

Mass gap for gravity localized on Weyl thick branes

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We consider thick brane configurations in a pure geometric Weyl integrable 5D space time, a non-Riemannian generalization of Kaluza–Klein (KK) theory involving a geometric scalar field. Thus, the 5D theory describes gravity coupled to a self-interacting scalar field which gives rise to the structure of the thick branes. We continue the study of the properties of a previously found family of solutions which is smooth at the position of the brane but involves naked singularities in the fifth dimension. Analyzing their graviton spectrum, we find that a particularly interesting situation arises for a special case in which the 4D graviton is separated from the KK gravitons by a mass gap. The corresponding effective Schrödinger equation has a modified Pöschl-Teller potential and can be solved exactly. Apart from the massless 4D graviton, it contains one massive KK bound state, and the continuum spectrum of delocalized KK modes. We also discuss the mass hierarchy problem, and explicitly compute the corrections to Newton’s law in the thin brane limit.

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I. INTRODUCTION

The success of brane world scenarios to address a number of high energy physics problems [1] and the expectation of possible experimental evidence for extra dimensions [2]–[7] have given rise to various modifications and generalizations of the initial thin brane models [4]–[7]. Among them we find several types of thick brane configurations: scalar smooth branes [8]–[10], tachyonic branes [11], branes on black holes [12, 13], branes with non-minimally coupled scalar fields [14, 15]; within this framework several physical aspects have been studied like localization of gravity [16]–[22] and matter fields [23], brane stability [24, 25], critical phenomena [26], etc.

The model to be considered here is of the class introduced by [8]. It describes Weyl gravity coupled to a geometrical scalar field and constitutes a non-compact generalization of the Randall–Sundrum (RS) model [5, 6] with the advantage that the 5D manifold is no longer restricted to be an orbifold and the branes in the action need not be introduced by hand.

In this paper, we briefly review localization of 4D gravity on previously constructed thick branes in a Weyl integrable manifold [21]. In Weyl geometry, a scalar field enters in the definition of the affine connection of the manifold, revealing its geometrical nature. Weyl integrable manifolds are invariant under Weyl rescalings (see, e.g., [20] for details); when this invariance is broken, via a self-interacting potential, for instance, the scalar field transforms into an observable degree of freedom.

Various classes of thick brane solutions have been discussed in the literature. In particular, in [8, 9, 16, 18] and [11, 15, 26], it was proposed to smooth out the results obtained in the RS model with the aid of a scalar field. Moreover, the stability of these thick objects was considered in [24]. All these models have a continuous KK graviton spectrum starting at zero mass, causing a need for explaining why such arbitrarily light extra gravitons have not led to detectable deviations from standard gravity. Although a number of mechanisms have been proposed how such light extra gravitons could go unobserved (see, e.g., [7]), it is obviously an interesting question whether thick brane solutions exist which have a mass gap and thereby avoid this problem altogether. Reconsidering some of the previously found families of solutions we have found one such case, and the main purpose of this paper is a detailed study of its properties.

The solution family in question involves a simple trigonometric warp factor [8, 9], [19]–[23]. Following [5, 8, 16], we study linear fluctuations of the classical metric and cast the wave equation for the transverse traceless modes in the form of a Schrödinger equation. Contrary to previous studies of this type of solutions, where the potential in this equation usually went to zero far from the brane, we fix the free parameter of the model in such a way that it asymptotes to a positive constant. The corresponding Schrödinger equation turns out to be exactly solvable, being of Pöschl–Teller type. Apart from the massless 4D graviton, it has one more bound state, describing a massive localized graviton, as well as a continuum tower of delocalized massive modes. Thus it indeed leads to a mass gap for the graviton spectrum, whose size is determined by the coupling constant of the scalar field self-interaction. This comes at a price, however: The curvature for this solution shows a naked singularity at a finite geodesic distance from the brane.

In this type of model, the massive modes give rise to

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small corrections to the Newton's law in 4D flat space time. This opens a new avenue to obtain theoretical predictions that can, in principle, be experimentally tested. Since in our special case we have all graviton wave functions in closed form, we are able to obtain the exact form of the correction to Newton's constant in the thin brane limit. Finally, we comment on the possible use of this configuration for the solution of the mass hierarchy problem along the lines of [5]–[7].

II. DEFINITION OF THE MODEL

Let us start with a pure geometrical Weyl action in five dimensions. This non-Riemannian generalization of the Kaluza–Klein theory is given by

$$S_5^W = \int_{M_5^W} \frac{d^5x \sqrt{|g|}}{16\pi G_5} e^{\frac{3}{2}\omega} [R + 3\tilde{\xi}(\nabla\omega)^2 + 6U(\omega)], \quad (1)$$

where M_5^W is a Weyl manifold specified by the pair (g_{MN}, ω) and ω is a scalar field. The Weylian Ricci tensor reads $R_{MN} = \Gamma_{MN,A}^A - \Gamma_{AM,N}^A + \Gamma_{MN}^P \Gamma_{PQ}^Q - \Gamma_{MQ}^P \Gamma_{NP}^Q$, where $\Gamma_{MN}^C = \{\Gamma_{MN}^C\} - \frac{1}{2}(\omega_{,M}\delta_N^C + \omega_{,N}\delta_M^C - g_{MN}\omega^{,C})$ are the affine connections on M_5^W , $\{\Gamma_{MN}^C\}$ are the Christoffel symbols and $M, N = 0, 1, 2, 3, 5$; ξ is an arbitrary coupling parameter, and $U(\omega)$ is a self-interacting potential for the scalar field ω .

By performing a conformal transformation $\hat{g}_{MN} = e^\omega g_{MN}$, the action (1) is mapped into the Einstein frame

$$S_5^R = \int_{M_5^R} \frac{d^5x \sqrt{|\hat{g}|}}{16\pi G_5} [\hat{R} + 3\xi(\hat{\nabla}\omega)^2 + 6\hat{U}(\omega)], \quad (2)$$

where the affine connections become Christoffel symbols, $\xi = \tilde{\xi} - 1$, $\hat{U} = e^{-\omega}U$ and hatted magnitudes refer to the Einstein frame. The corresponding field equations read

$$\begin{aligned} \hat{R}_{MN} = & -\left(3\xi\hat{\nabla}_M\omega\hat{\nabla}_N\omega + 2\hat{U}\hat{g}_{MN}\right), \\ \hat{\nabla}^2\omega = & \frac{1}{\xi}\frac{d\hat{U}}{d\omega}. \end{aligned} \quad (3)$$

In order to get solutions to the theory (1) that preserve 4D Poincaré invariance we consider the following ansatz

$$ds_5^2 = e^{2A(y)}\eta_{mn}dx^m dx^n + dy^2, \quad (4)$$

where $e^{2A(y)}$ is the warp factor depending on the extra coordinate y , and $m, n = 0, 1, 2, 3$. Thus, the field equations (3) adopt the form [32]

$$\begin{aligned} \omega'' + 4A'\omega' + \frac{3}{2}(\omega')^2 = & \frac{1}{\xi}\frac{d\hat{U}}{d\omega}e^\omega, \\ A'' + 4(A')^2 + \frac{3}{2}A'\omega' = & \frac{1}{2}\left(4\hat{U} - \frac{1}{\xi}\frac{d\hat{U}}{d\omega}\right)e^\omega. \end{aligned} \quad (5)$$

Solutions to these equations can be obtained by imposing the condition $\omega = 2kA$ and following the conformal technique described in [19, 20]. In this work we shall reconsider the concrete family of solutions [8, 9],[19]–[21]

$$\begin{aligned} e^{2A(y)} = & \left[\cos\left(\sqrt{8\lambda b}(y-c)\right)\right]^{\frac{3}{2b}}, \\ e^{\omega(y)} = & \left[\cos\left(\sqrt{8\lambda b}(y-c)\right)\right]^{-\frac{2}{b}}, \end{aligned} \quad (6)$$

where $\hat{U} = \lambda e^{16\xi\omega}$, $b = 1 + 16\xi$, c is an arbitrary integration constant and λ is a coupling constant. This solution corresponds to the self-interacting potential $U = \lambda e^{(1+16\xi)\omega}$ in the Weyl frame.

The ξ parameter determines the nature of the scalar field. In particular, for positive values of ξ ($b > 1$), the scalar field ω turns out to be a phantom (ghost), as is seen from (2). If ξ is negative ($b < 1$), ω is a conventional scalar field.

In [21] it was shown that for $\lambda, b > 0$ and $b \neq \frac{15}{8}$ the 5D curvature scalar R_5 is singular at $\sqrt{8\lambda b}(y-c) = \pm\frac{\pi}{2}$, so that the domain of the extra coordinate should be chosen as the interval $-\frac{\pi}{2} \leq \sqrt{8\lambda b}(y-c) \leq \frac{\pi}{2}$. These curvature singularities are naked ones, as one can easily show analytically.

III. PERTURBING THE METRIC

Following [5, 8], let us study the metric fluctuations h_{mn} of the metric (4) given by the perturbed line element

$$ds_5^2 = e^{2A(y)}[\eta_{mn} + h_{mn}(x, y)]dx^m dx^n + dy^2. \quad (7)$$

One must consider the fluctuations of the scalar field when treating the ones of the metric since they are coupled. However, in [8] it was shown that the transverse traceless modes of the metric fluctuations h_{mn}^T decouple from the scalar sector, allowing us to study the dynamics of these fluctuations analytically in closed form. Following [6]–[9] we perform the coordinate transformation

$$dw = e^{-A}dy. \quad (8)$$

This leads to a conformally flat metric and to the following wave equation for the transverse traceless modes of the metric perturbations,

$$(\partial_w^2 + 3A'\partial_w + \square^\eta)h_{mn}^T = 0, \quad (9)$$

where \square^η is the standard wave operator in four dimensions. Due to the unbroken Poincaré invariance, this equation supports a massless and normalizable 4D graviton given by $h_{mn}^T = C_{mn}e^{ipx}$, where C_{mn} are constant parameters and $p^2 = m^2 = 0$. Using the ansatz $h_{mn}^T = e^{ipx}e^{-3A/2}\Psi_{mn}(w)$ the wave equation (9) can be recast in the form of a Schrödinger's equation

$$[\partial_w^2 - V(w) + m^2]\Psi = 0. \quad (10)$$

Here we have dropped the subscripts on Ψ , m is the mass of the KK excitation, the potential reads

$$V(w) = \frac{3}{2}\partial_w^2 A + \frac{9}{4}(\partial_w A)^2, \quad (11)$$

and is fully defined by the curvature of the 5D space-time.

The question which we wish to study now is, are there values of the parameter b in the two-parameter family of solutions (6) such that (i) the transformation (8) is decompactifying, i.e. maps the compact y -interval onto the real w -line and (ii) the resulting Schrödinger equation (10) shows a mass gap between the massless (physical) graviton solution and the massive KK gravitons.

It is easily seen that (i) requires us to restrict $b \leq 3/4$. Proceeding to (ii), we face the difficulty that for generic values of b the differential equation (8) does not allow us to construct the function $w(y)$ in explicit form. However, we can still use (8) and (11) to calculate the potential as a function of y :

$$V(w(y)) = 9\lambda \left[\cos(\sqrt{8\lambda b}(y-c)) \right]^{\frac{3}{2b}} \times \left[-\frac{15}{8b} + \left(\frac{15}{8b} - 1 \right) \frac{1}{\cos^2(\sqrt{8\lambda b}(y-c))} \right] \quad (12)$$

Moreover, the zero-mass solution for the potential (11) is the ground state, and is given simply by [16]

$$\Psi_0(w) = C_0 e^{3A(w)/2} \quad (13)$$

as can be verified using (11) (C_0 is a normalization constant). From (8), $w(y)$ is a monotonous function, so that

$$\lim_{w \rightarrow \pm\infty} V(w) = \lim_{\sqrt{8\lambda b}(y-c) \rightarrow \pm\frac{\pi}{2}} V(w(y)). \quad (14)$$

From (12) this limit is zero for $b < 3/4$, while for the borderline case $b = 3/4$ it approaches the finite value $V_\infty = 27\lambda/2$. This is sufficient to exclude a mass gap for $b < 3/4$, since the Schrödinger equation (10) will have a continuous spectrum starting at zero if the potential goes to zero at infinity.

Thus, in the following we will focus on the case $b = 3/4$. Here (8) can be solved explicitly, yielding

$$\cos[\sqrt{6\lambda}(y-c)] = \text{sech}[\sqrt{6\lambda}(w-w_0)]. \quad (15)$$

The function $A(w)$ takes the form

$$A(w) = \ln \left\{ \text{sech} \left[\sqrt{6\lambda}(w-w_0) \right] \right\}, \quad (16)$$

and the potential becomes a modified Pöschl-Teller one:

$$V(w) = \frac{9\lambda}{2} \left\{ 3 - 5 \text{sech}^2 \left[\sqrt{6\lambda}(w-w_0) \right] \right\}. \quad (17)$$

Since this potential asymptotically reaches a positive value, we automatically get a mass gap for this case, allowing for a phenomenological solution of the dangerous light KK excitations problem (similar results have

been obtained for de Sitter [13, 27] and non standard [28] branes). The corresponding Schrödinger equation (10) turns, after a rescaling $u = \sqrt{6\lambda}(w-w_0)$, into

$$\left[\partial_u^2 + \frac{15}{4} \text{sech}^2(u) + \frac{m^2}{6\lambda} - \frac{9}{4} \right] \Psi = 0. \quad (18)$$

In this standardized form it is the special case $n = 3/2$ of the well-known family of Schrödinger equations

$$\left[-\partial_u^2 - n(n+1) \text{sech}^2(u) \right] \Psi = E \Psi. \quad (19)$$

This implies (see, e.g., [29]) that in this potential there are two bound states, the ground state Ψ_0 with energy $E_0 = -n^2 = -9/4$ and one excited state Ψ_1 with energy $E_1 = -(n-1)^2 = -1/4$. The ground state wave function is

$$\Psi_0(w) = C_0 \text{sech}^{3/2}(w). \quad (20)$$

This is just the zero-mass state (13), confirming the absence of tachyonic modes with $m^2 < 0$. The wave function of the excited state is

$$\Psi_1(w) = C_1 \sinh(w) \text{sech}^{3/2}(w). \quad (21)$$

It represents a massive graviton of mass $m^2 = 12\lambda$ localized on the brane. The continuous spectrum starts at $E = 0$, corresponding to

$$m^2 \geq \frac{27}{2}\lambda. \quad (22)$$

These states asymptotically turn into plane waves, and represent delocalized KK massive gravitons. Their explicit expressions can be given in terms of the associated Legendre functions of the first kind P_ν^μ , of degree $\nu = 3/2$ and purely imaginary order $\mu = i\rho$:

$$\Psi^\mu(w) = \sum_{\alpha=\pm} C_\alpha P_{\frac{3}{2}}^{\alpha i\rho} \left(\tanh \left[\sqrt{6\lambda}(w-w_0) \right] \right) \quad (23)$$

where C_\pm are integration constants and

$$\rho = |\mu| = \sqrt{\frac{m^2}{6\lambda} - \frac{27}{12}}. \quad (24)$$

It should be mentioned that the wave functions of these KK gravitons are, in the original y coordinates, singular at the endpoints $y = c \pm \frac{\pi}{2\sqrt{6\lambda}}$.

To study the brane content of the $b = 3/4$ solution we calculate the energy density. In the w coordinates this is

$$\begin{aligned} T_{00}(w) &= -\frac{3}{8\pi G_5} [\partial_w^2 A + (\partial_w A)^2] \\ &= \frac{9\lambda}{4\pi G_5} \left(\frac{2}{\cosh^2(\sqrt{6\lambda}w)} - 1 \right) \end{aligned} \quad (25)$$

In the limit of large λ this becomes

$$T_{00} \simeq \frac{9}{\pi G_5} \left(\sqrt{\frac{\lambda}{6}} \delta(w) - \frac{\lambda}{4} \right) \quad (26)$$

Thus, up to a cosmological constant term, the energy density localizes at a single brane located at the center, $w = 0$. For the applications presented below the constant term is not relevant, since it can be removed by a renormalization of the vacuum energy. Finally, it should be mentioned that in the w coordinates the curvature singularities are at $w = \pm\infty$, but still at a finite geodesic distance from the brane.

IV. PHYSICAL APPLICATIONS

To apply our brane configuration in the context of [5, 6, 7], first we need to establish the connection between the Planck scales in four and five dimensions. In order to get the four-dimensional effective Planck scale, we replace the Minkowski metric by a four-dimensional metric \bar{g}_{mn} in eq. (4), leading to an effective four-dimensional action. We look at the curvature term from which one can derive the scale of the gravitational interactions

$$S_{eff} \supset \int d^4x \int_0^\infty dw 2M_*^3 \sqrt{|g|} e^{\frac{3}{2}\omega} e^{3A} \bar{R}, \quad (27)$$

where we performed the coordinate transformation (8), M_* is the Planck scale in five dimensions, and \bar{R} is the four-dimensional Ricci scalar. Because the low energy fluctuations have no w dependence, we can perform the direct integration along w , obtaining a purely four-dimensional action, and calculate the four-dimensional effective Planck scale

$$M_{pl}^2 = 2M_*^3 \int_0^\infty dw e^{\frac{3}{2}\omega} e^{3A}. \quad (28)$$

For our particular solution, eq. (6) with $b = 3/4$, we get for large r

$$\int_0^r dw e^{\frac{3}{2}\omega} e^{3A} \approx \frac{1}{\sqrt{6\lambda}} \left(\frac{5\pi}{32} - \frac{128}{7} e^{-7\sqrt{6\lambda}r} \right), \quad (29)$$

so that

$$M_{pl}^2 = \frac{5\pi}{16\sqrt{6\lambda}} M_*^3. \quad (30)$$

Thus M_{pl} is finite, and we also see that it depends only weakly on r for $r \rightarrow \infty$.

A. KK corrections to Newton's law

The massive gravitons give rise to corrections to Newton's law in ordinary 4D flat spacetime. In the present

model and in the thin brane limit, they can be expressed in the following way [6, 16]:

$$U(r) \sim \frac{M_1 M_2}{r} \left[G_4 + M_*^{-3} e^{-m_1 r} |\hat{\Psi}_1(w_0)|^2 + M_*^{-3} \int_{m_0}^\infty dm e^{-mr} |\hat{\Psi}^{\mu(m)}(w_0)|^2 \right]. \quad (31)$$

Here the thin brane represents the physical universe, located at $w = w_0$ in the extra dimension. G_4 denotes the four dimensional gravitational coupling, $\hat{\Psi}_1$ is the wave function of our single normalizable excited state, and $\hat{\Psi}^{\mu(m)}$ denotes the continuous eigenfunctions which have to be integrated over their masses, and (except for $\mu = 0$) to be summed over the two independent eigenfunctions. From (23) those can be chosen as (setting $w_0 = 0$ for convenience)

$$\hat{\Psi}_\pm^\mu(w) = C_\pm(\rho) P_{\frac{3}{2}}^{\pm i\rho} \left(\tanh(\sqrt{6\lambda}w) \right). \quad (32)$$

The 'hat' on $\hat{\Psi}_1$ means that it is normalized, while for the Ψ^μ , which asymptotically for large $|w|$ are of plane wave form, it means normalization in the plane wave sense. The thin brane limit has been used to set both test bodies to the center of the brane in the transverse direction, $w = 0$. Since $\hat{\Psi}_1(w)$ is an odd function it does not contribute in this limit. To calculate the corrections due to the continuous modes, first we have to compute the normalization constants $C_\pm(\rho)$. Using the large x approximation of the $\tanh(x)$,

$$\tanh(x) \simeq 1 - 2e^{-2x} \quad (33)$$

and Eq. (8) of section 3.9.2 of [30], one can easily show that for $w \rightarrow \infty$

$$P_{\frac{3}{2}}^{\pm i\rho} \left(\tanh(\sqrt{6\lambda}w) \right) \sim \frac{1}{\Gamma(1 \mp i\rho)} e^{\pm i\sqrt{6\lambda}\rho w}. \quad (34)$$

Since $|\Gamma(1 - i\rho)| = |\Gamma(1 + i\rho)|$ this yields

$$C_+(\rho) = C_-(\rho) = \frac{|\Gamma(1 + i\rho)|}{\sqrt{2\pi}} \quad (35)$$

Further, we need the value of $|\hat{\Psi}^\mu(0)|$. From [30]

$$P_\nu^\mu(0) = \frac{2^\mu}{\sqrt{\pi}} \cos \left[\frac{1}{2} \pi (\nu + \mu) \right] \frac{\Gamma \left(\frac{1}{2} + \frac{1}{2} \nu + \frac{1}{2} \mu \right)}{\Gamma \left(1 + \frac{1}{2} \nu - \frac{1}{2} \mu \right)} \quad (36)$$

For $\mu = \pm i\rho$ and taking the absolute value this can be simplified to

$$|P_{\frac{3}{2}}^{\pm i\rho}(0)| = \frac{\sqrt{\pi}}{\left| \Gamma \left(-\frac{1}{4} + \frac{i\rho}{2} \right) \right| \left| \Gamma \left(\frac{7}{4} + \frac{i\rho}{2} \right) \right|}. \quad (37)$$

Putting things together, we can express the content of the square brackets in (31) as $G_4 + \Delta G_4$, where

$$\Delta G_4 = M_*^{-3} \int_{m_0}^\infty dm e^{-mr} \left| \frac{\Gamma(1 + i\rho)}{\Gamma \left(-\frac{1}{4} + \frac{i\rho}{2} \right) \Gamma \left(\frac{7}{4} + \frac{i\rho}{2} \right)} \right|^2. \quad (38)$$

For the calculation of this integral, it will be convenient to change variables from m to ρ . This yields

$$\Delta G_4 = M_*^{-3} \sqrt{6\lambda} \int_0^\infty \frac{d\rho}{\sqrt{1 + \frac{27}{12\rho^2}}} e^{-r\sqrt{6\lambda}\sqrt{\rho^2 + \frac{27}{12}}} \times \left| \frac{\Gamma(1+i\rho)}{\Gamma(-\frac{1}{4} + \frac{i\rho}{2})\Gamma(\frac{7}{4} + \frac{i\rho}{2})} \right|^2. \quad (39)$$

Although it seems not possible to do this integral in closed form, it can be easily calculated numerically as a function of $\sqrt{6\lambda}r$. Moreover, in the thin brane limit $\lambda \rightarrow \infty$ the integral will be dominated by the small - ρ region, so that it can be well-approximated by expanding the prefactor of the exponential at $\rho = 0$. In this way one obtains for the thin brane limit

$$\Delta G_4 \sim M_*^{-3} \frac{1}{\left| \Gamma(-\frac{1}{4})\Gamma(\frac{7}{4}) \right|^2} \frac{e^{-\sqrt{\frac{27}{2}}\lambda r}}{r} (1 + O(\frac{1}{r\sqrt{\lambda}})). \quad (40)$$

We remark that, away from the thin brane limit, the correction would also involve the massive bound state. The form of Newton's law would then become rather involved, and depend on the precise location of the two test bodies along the fifth dimension. Further corrections to Newton's law might be induced by the scalar field modes, whose spectrum we have not attempted to analyze here. Corrections to Newton's law were also obtained for brane models with a non-minimally coupled bulk scalar field in [15].

B. Solution of the mass hierarchy problem

Our special configuration also fits well into the framework of [7], where it was shown how to arrive at a solution of the mass hierarchy problem using a non-compact extra dimension. First, we observe that the warp factor (16) in the stiff limit reproduces the Randall-Sundrum one:

$$e^{2A} \simeq 4e^{-2\sqrt{6\lambda}|w|}. \quad (41)$$

In the set-up of [7], the limiting thin brane at $w = 0$ does not represent the physical brane, but a hidden “Planck brane”. The physical brane will be located at a distance w , where the effective Planck scale is reduced from $M_{Pl} = 10^{19} GeV$ by the factor (41). By an appropriate choice of w this exponential factor can be made to account for the unwanted discrepancy between the Planck and electroweak scales. Using our special brane configuration, which has a mass gap proportional to $\sqrt{\lambda}$, we can implement this solution of the hierarchy problem without having to face possible problems with low-mass KK graviton excitations. However, this issue will be studied in more detail in [31] in a Riemannian manifold.

V. CONCLUDING REMARKS

Continuing previous work by two of the authors [21] on Weyl thick branes, we have performed an analysis of the transverse traceless modes of the linear fluctuations of a family of classical backgrounds. For a particular case, we have arrived at an exact solution of the effective Schrödinger equation obeyed by these modes. This is exceptional, since usually for smooth brane configurations with Poincaré symmetry one is not able to integrate the Schrödinger equation for the massive modes explicitly (a toy model with calculable KK modes was given in [16]). More importantly, the resulting graviton spectrum is unusual in that it shows both a mass gap and a continuum of massive KK modes, as well as a single massive localized graviton state. The mass gap is proportional to the scalar self-interaction coupling constant $\sqrt{\lambda}$. Its existence provides an easy way to control the excitation of the KK gravitons and concomitant energy loss into the fifth dimension. This suppression of the delocalized gravitons should also render harmless the presence of naked singularities at $w = \pm\infty$. We have given two applications: using this configuration along the lines of RS [6] we have, in the thin brane limit, obtained the correction to Newton's law in closed form; only the massive modes contribute in this limit. In the framework of [7], our configuration leads to a solution of the hierarchy problem without low-energy KK graviton excitations.

It is worth noticing that one can start with a setup free of phantom fields in the Einstein frame from the very beginning as in [8, 9, 22], consider solutions for the warp factor of the form $e^{2A} = \cos^b(ay)$ (with a suitable self-interacting potential) and properly set the value for the parameter $b = 2$ in order to ensure the appearance of the mass gap [31]. To the best of our knowledge, only the non-compact version of this solution $e^{2A} = \cosh^b(ay)$ has been studied in the literature.

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